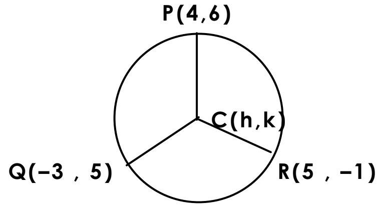
CIRCLE KE SOLUTIONS

01. Find equation of circle passing through (4 , 6) ; (−3 , 5) & (5 , −1)

Solution :



Step 1 :

CP = CQ

$CP^2 = CQ^2$

$(h-4)^2 + (k-6)^2 = (h+3)^2 + (k-5)^2$

$h^2 - 8h + 16 + k^2 - 12k + 36 = h^2 + 6h + 9 + k^2 - 10k + 25$

$-8h - 12k + 52 = 6h - 10k + 34$

$18 = 14h + 2k$

$9 = 7h + k$ ……………… (1)

Step 2 :

CP = CR

$CP^2 = CR^2$

$(h-4)^2 + (k-6)^2 = (h-5)^2 + (k+1)^2$

$h^2 - 8h + 16 + k^2 - 12k + 36 = h^2 - 10h + 25 + k^2 + 2k + 1$

$-8h - 12k + 52 = -10h + 2k + 26$

$2h - 14k = -26$

$h - 7k = -13$ ………………… (2)

Step 3 : 7 x $7h + k = 9$; $h - 7k = -13$

$49h + 7k = 63$

$h - 7k = -13$

$50h = 50$

$h = 1$

$k = 2$ $\quad C \equiv (1 , 2)$

Step 4 : $r = CP$; C (1 , 2) , P(4,6)

$r^2 = CP^2$

$= (1-4)^2 + (2-6)^2$

$= 25$

Step 5 : C(1 , 2) , $r^2 = 25$

$(x-h)^2 + (y-k)^2 = r^2$

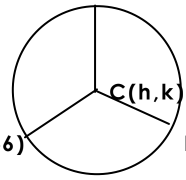
$(x-1)^2 + (y-2)^2 = 25$

$x^2 - 2x + 1 + y^2 - 4y + 4 = 25$

$x^2 + y^2 - 2x - 4y - 20 = 0$

02. **Find equation of circle passing through (4 , 1) ; (−3 , −6) & (−2 , 1)**

Solution :



Step 1 :

$CP = CQ$

$CP^2 = CQ^2$

$(h-4)^2 + (k-1)^2 = (h+3)^2 + (k+6)^2$

$h^2 - 8h + 16 + k^2 - 2k + 1 = h^2 + 6h + 9 + k^2 + 12k + 36$

$-8h - 2k + 17 = 6h + 12k + 45$

$-28 = 14h + 14k$

$-2 = h + k$ ……………… (1)

Step 2 :

$CP = CR$

$CP^2 = CR^2$

$(h-4)^2 + (k-1)^2 = (h+2)^2 + (k-1)^2$

$h^2 - 8h + 16 + k^2 - 2k + 1 = h^2 + 4h + 4 + k^2 - 2k + 1$

$-8h - 2k + 17 = 4h - 2k + 5$

$12 = 12h$

$h = 1$ ………………… (2)

Step 3 : sub $h = 1$ in (1)

$k = -3$ $C \equiv (1 , -3)$

Step 4 : $r = CP$; $C (1 , -3)$, $P(4,1)$

$r^2 = CP^2$

$= (1-4)^2 + (-3-1)^2$

$= 25$

Step 5 : $C(1 , -3)$, $r^2 = 25$

$(x-h)^2 + (y-k)^2 = r^2$

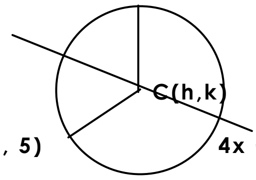
$(x-1)^2 + (y+3)^2 = 25$

$x^2 - 2x + 1 + y^2 + 6y + 9 = 25$

$x^2 + y^2 - 2x + 6y - 15 = 0$

03. Find equation of circle passing through (4 , 1) ; (6 , 5) & whose center lies on $4x + y = 16$

Solution :



Step 1 :

Since C(h , k) lies on $4x + y = 16$

$4h + k = 16$ ……………… (1)

Step 2 :

$CP = CQ$

$CP^2 = CQ^2$

$(h - 4)^2 + (k - 1)^2 = (h - 6)^2 + (k - 5)^2$

$h^2 - 8h + 16 + k^2 - 2k + 1 = h^2 - 12h + 36 + k^2 - 10k + 25$

$-8h - 2k + 17 = -12h - 10k + 61$

$4h + 8k = 44$

$h + 2k = 11$ ………………… (2)

Step 3 : $2 \times \quad 4h + k = 16$

$h + 2k = 11$

$8h + 2k = 32$

$h + 2k = 11$

$7h = 21$

$h = 3$

$k = 4$ $\quad C \equiv (3 , 4)$

Step 4 : $r = CP$; C (3 , 4) , P(4,1)

$r^2 = CP^2$

$= (3 - 4)^2 + (4 - 1)^2$

$= 10$

Step 5 : C(3 , 4) , $r^2 = 10$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 3)^2 + (y - 4)^2 = 10$

$x^2 - 6x + 9 + y^2 - 8y + 16 = 10$

$x^2 + y^2 - 6x - 8y + 15 = 0$

04. Find equation of circle passing through (1 , −4) ; (5 , 2) & whose center lies on $x - 2y + 9 = 0$

ans : $x^2 + y^2 + 6x - 6y - 47 = 0$

05. find equation of the circle touching both axes and passing through (1,2)

Step 1 : $CP = r$

$CP^2 = r^2$

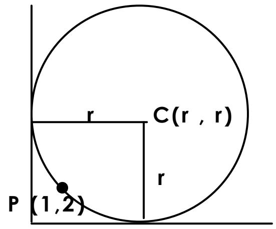
$(r - 1)^2 + (r - 2)^2 = r^2$

$r^2 - 2r + 1 + r^2 - 4r + 4 = r^2$

$r^2 - 6r + 5 = 0$

$(r - 1)(r - 5) = 0$

$r = 1 \quad ; \quad r = 5$



Step 2 : $r = 1 \quad ; \quad C(1,1)$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 1)^2 + (y - 1)^2 = 1$

$x^2 - 2x + 1 + y^2 - 2y + 1 = 1$

$x^2 + y^2 - 2x - 2y + 1 = 0$

$r = 5 \quad ; \quad C(5,5)$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 5)^2 + (y - 5)^2 = 25$

$x^2 - 10x + 25 + y^2 - 10y + 25 = 25$

$x^2 + y^2 - 10x - 10y + 25 = 0$

06. find equation of the circle touching both axes and passing through (−9,8)

Step 1 : $CP = r$

$CP^2 = r^2$

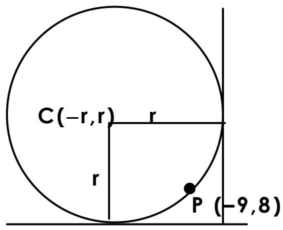
$(-r + 9)^2 + (r - 8)^2 = r^2$

$r^2 - 18r + 81 + r^2 - 16r + 64 = r^2$

$r^2 - 34r + 145 = 0$

$(r - 5)(r - 29) = 0$

$r = 5 \quad ; \quad r = 29$



Step 2 : $r = 5 \quad ; \quad C(-5,5)$

$(x - h)^2 + (y - k)^2 = r^2$

$(x + 5)^2 + (y - 5)^2 = 25$

$x^2 + 10x + 25 + y^2 - 10y + 25 = 25$

$x^2 + y^2 + 10x - 10y + 25 = 0$

$r = 29 \quad ; \quad C(-29,29)$

$(x - h)^2 + (y - k)^2 = r^2$

$(x + 29)^2 + (y - 29)^2 = 841$

$x^2 + 58x + 841 + y^2 - 58y + 841 = 841$

$x^2 + y^2 + 58x - 58y + 841 = 0$

LIMITS KE SOLUTIONS

01. $$\lim_{x \to 3} \frac{\log x - \log 3}{x - 3}$$

$x = 3 + h$

$$= \lim_{h \to 0} \frac{\log(3 + h) - \log 3}{3 + h - 3}$$

$$= \lim_{h\to 0} \frac{\log\left(\frac{3 + h}{3}\right)}{h}$$

$$= \lim_{h\to 0} \frac{\log\left(1 + \frac{h}{3}\right)}{h}$$

$$= \lim_{h\to 0} \frac{1}{h} \log\left(1 + \frac{h}{3}\right)$$

$$= \lim_{h\to 0} \log\left(1 + \frac{h}{3}\right)^{\frac{1}{h}}$$

$$= \lim_{h\to 0} \log\left[\left(1 + \frac{h}{3}\right)^{\frac{3}{h}}\right]^{\frac{1}{3}}$$

$$= \log e^{\frac{1}{3}}$$

$$= \frac{1}{3} \log e$$

$$= \frac{1}{3}$$

02. $$\lim_{x \to 5} \frac{\log x - \log 5}{x^2 - 25}$$

$$= \lim_{x \to 5} \frac{\log x - \log 5}{x - 5} \cdot \frac{1}{x + 5}$$

$x = 5 + h$

$$= \lim_{h \to 0} \frac{\log(5 + h) - \log 5}{5 + h - 5} \cdot \frac{1}{5 + h + 5}$$

$$= \lim_{h\to 0} \frac{\log\left(\frac{5 + h}{5}\right)}{h} \cdot \frac{1}{10 + h}$$

$$= \lim_{h\to 0} \frac{\log\left(1 + \frac{h}{5}\right)}{h} \cdot \frac{1}{10 + h}$$

$$= \lim_{h\to 0} \frac{1}{h} \log\left(1 + \frac{h}{5}\right) \cdot \frac{1}{10 + h}$$

$$= \lim_{h\to 0} \log\left(1 + \frac{h}{5}\right)^{\frac{1}{h}} \cdot \frac{1}{10 + h}$$

$$= \lim_{h\to 0} \log\left(1 + \frac{h}{5}\right)^{\frac{5}{h} \cdot \frac{1}{5}} \cdot \frac{1}{10 + h}$$

$$= \log e^{\frac{1}{5}} \cdot \frac{1}{10 + 0}$$

$$= \frac{1}{5} \log e \cdot \frac{1}{10}$$

$$= \frac{1}{50}$$

03. $$\lim_{x\to 3} (x - 2)^{\frac{1}{x - 3}}$$

$x = 3 + h$

$$= \lim_{h\to 0} (3 + h - 2)^{\frac{1}{3 + h - 3}}$$

$$= \lim_{h\to 0} (1 + h)^{\frac{1}{h}}$$

$$= e$$

ELLIPSE KE SOLUTIONS

Q2 18. distance between directrices is 10 and passing through $(-\sqrt{5}, 2)$. Find equation of the ellipse

Solution

Let the equation of the ellipse : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

distance between directrices = 10

$$2\frac{a}{e} = 10$$

$$\frac{a}{e} = 5$$

$$e = \frac{a}{5}$$

passing through $(-\sqrt{5}, 2)$

$$\frac{5}{a^2} + \frac{4}{b^2} = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

Now

$$b^2 = a^2(1 - e^2)$$

$$b^2 = a^2\left(1 - \frac{a^2}{25}\right)$$

$$b^2 = a^2\left(\frac{25 - a^2}{25}\right) \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

subs in 1

$$\frac{5}{a^2} + \frac{4}{\frac{a^2(25 - a^2)}{25}} = 1$$

$$\frac{5}{a^2} + \frac{100}{a^2(25 - a^2)} = 1$$

$$\frac{5(25 - a^2) + 100}{a^2(25 - a^2)} = 1$$

$$5(25 - a^2) + 100 = a^2(25 - a^2)$$

$$125 - 5a^2 + 100 = 25a^2 - a^4$$

$$a^4 - 30a^2 + 225 = 0$$

$$(a^2 - 15)(a^2 - 15) = 0$$

$$a^2 = 15$$

subs in (2) $\quad b^2 = \frac{15(25 - 15)}{25} = 6$

hence the equation of the ellipse : $\frac{x^2}{15} + \frac{y^2}{6} = 1$

NM – MITHIBAI MARCH 2013 :

Q : Find equation of the ellipse whose major axis is y axis and minor axis is the x – axis if the distances between foci is $2\sqrt{5}$ and passing through $(\sqrt{2}, 3/\sqrt{2})$

Solution

Let the equation of the ellipse : $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$; $b > a$

$$\text{distance between foci} = 2\sqrt{5}$$

$$2\ be = 2\sqrt{5}$$

$$be = \sqrt{5}$$

$$e = \frac{\sqrt{5}}{b}$$

passing through $(\sqrt{2}, 3/\sqrt{2})$

$$\frac{2}{a^2} + \frac{9}{2b^2} = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

Now

$$a^2 = b^2(1 - e^2)$$

$$a^2 = b^2\left(1 - \frac{5}{b^2}\right)$$

$$a^2 = b^2\left(\frac{b^2 - 5}{b^2}\right)$$

$$a^2 = b^2 - 5 \ldots\ldots\ldots\ldots\ldots (2)$$

subs in 1

$$\frac{2}{b^2 - 5} + \frac{9}{2b^2} = 1$$

$$\frac{4b^2 + 9b^2 - 45}{2b^2(b^2 - 5)} = 1$$

$$13b^2 - 45 = 2b^4 - 10b^2$$

$$2b^4 - 23b^2 + 45 = 0$$

$$2b^4 - 18b^2 - 5b^2 + 45 = 0$$

$$2b^2(b^2 - 9) - 5(b^2 - 9)$$

$b^2 = 9$; $b^2 = \dfrac{5}{2}$

$a^2 = b^2 - 5$; $a^2 = b^2 - 5$

$= 9 - 5$; $a^2 = -5/2$ ……… discard

$a^2 = 4$

hence equation is : $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$